An Improved Combinatorial Algorithm for Boolean Matrix Multiplication

Huacheng Yu

Stanford University

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Introduction

Boolean Matrix Multiplication (Boolean MM or BMM):

$$\begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 1 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & \cdots & 1 \\ 1 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix} =$$

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- This paper [2015]: $\hat{O}(n^3/\log^4 n)$

Algebraic vs Combinatorial

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Algebraic Algorithms	Combinatorial Algorithms
Asymptotically faster	Asymptotically slower

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Algebraic Algorithms	Combinatorial Algorithms
Asymptotically faster	Asymptotically slower
Difficult to implement slow in practice	Very simple fast in practice
Require similar algebraic structure to generalize	Generalizable in a different way

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$$\text{Triangle detection} \xleftrightarrow{\text{“sub-cubic”}} \text{Boolean MM}$$

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$$O(n^3/g(n)) \xrightarrow{\text{combinatorial}} O(n^3/g(n^{1/3}))$$

Main Result

Our main theorem:

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Given a graph G *on* n *vertices*

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Given a graph G on n vertices, we can detect if there is a triangle in G using a combinatorial algorithm in $\hat{O}(n^3/\log^4 n)$ time.

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A general framework for triangle detection:

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If there is an algorithm that takes a graph and can "efficiently" find and solve triangle detection on a "large" subgraph, then triangle detection is "easy" in general.

Triangle Detection Algorithm

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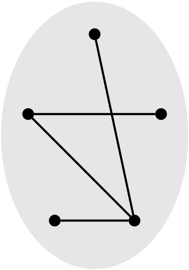

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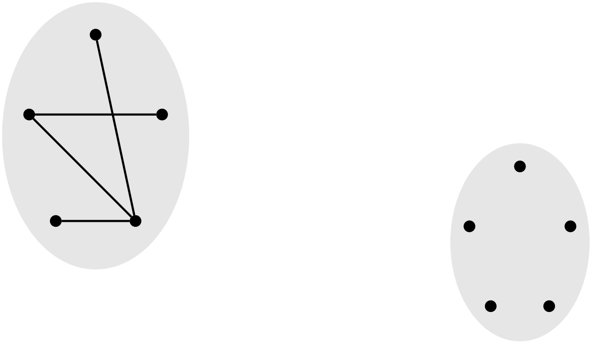

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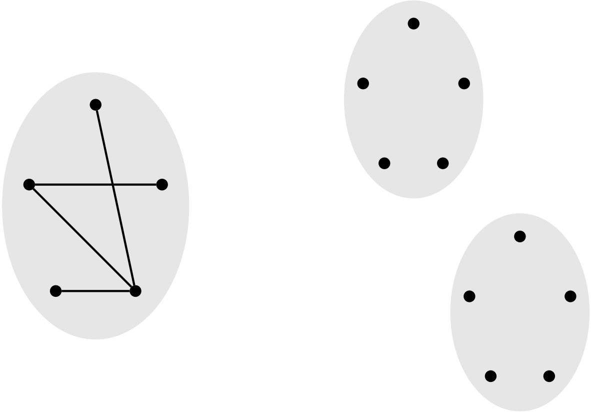

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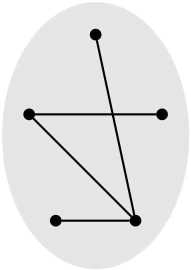

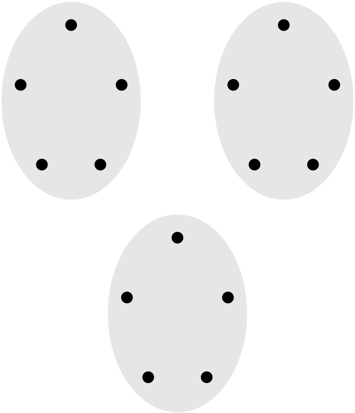

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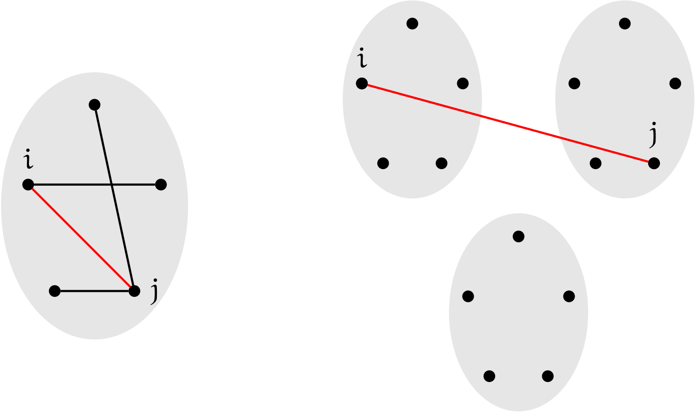

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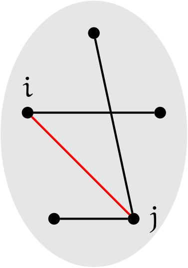

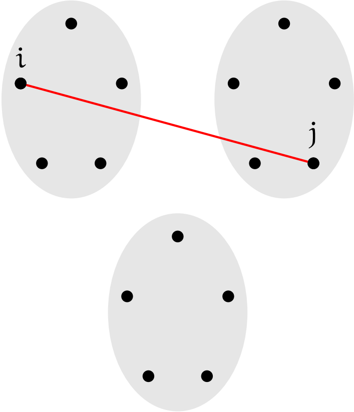

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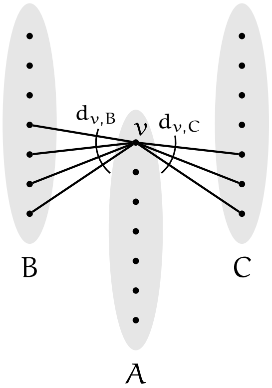

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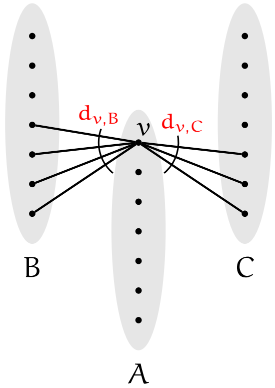

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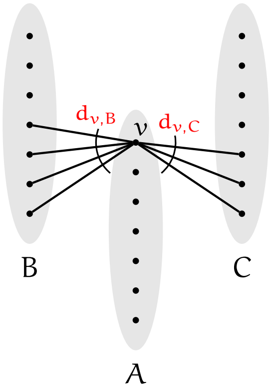

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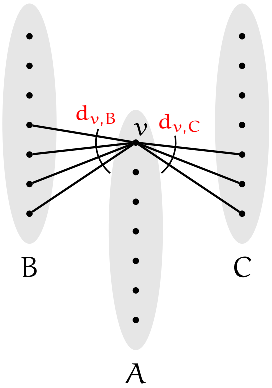

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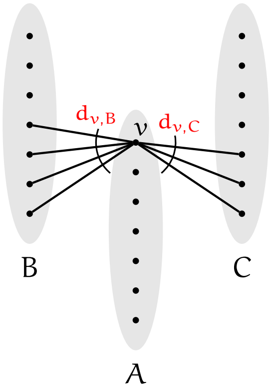
- **recursion!** (also used in Chan's algorithm)



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The sparse case: $d_{v,B} d_{v,C} \leqslant \hat{O}(n^2/\log^2 n)$ for every $v \in A$.

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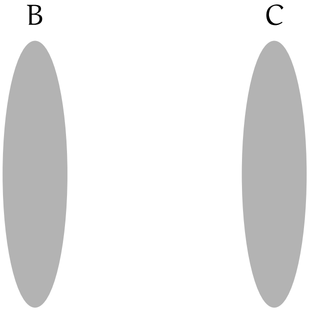

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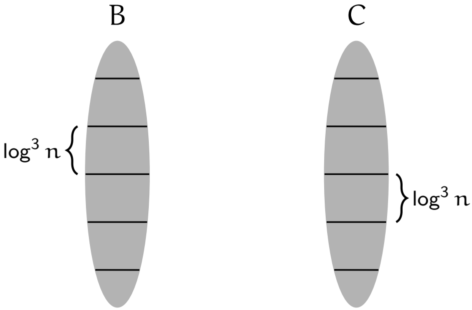

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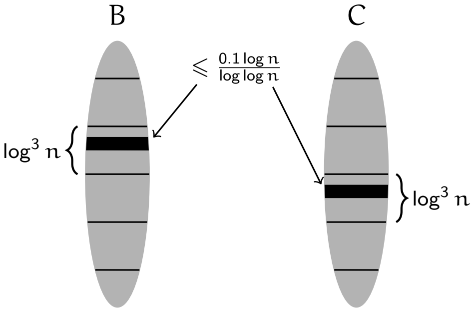

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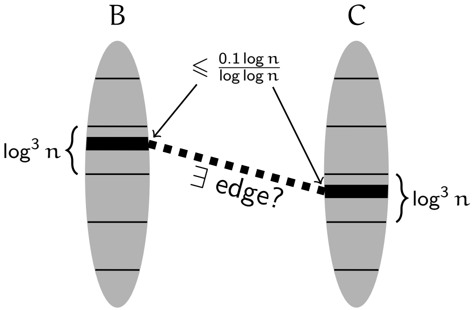

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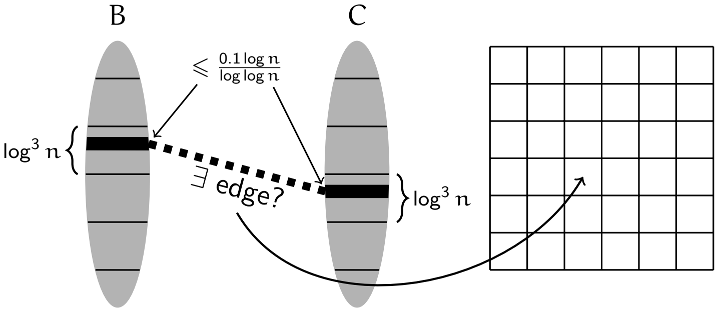

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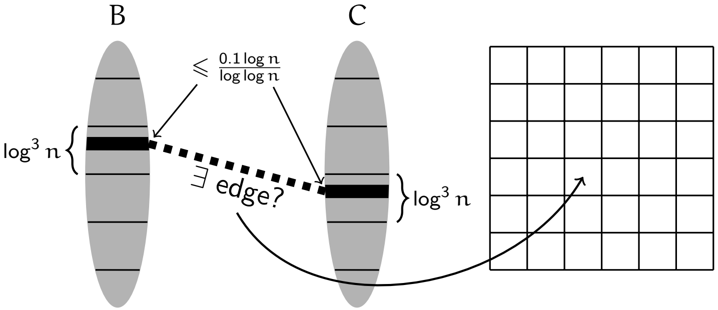

Preprocessing: $n^2(\log^3 n)^{0.2 \log n / \log\log n} = O(n^{2.6})$.

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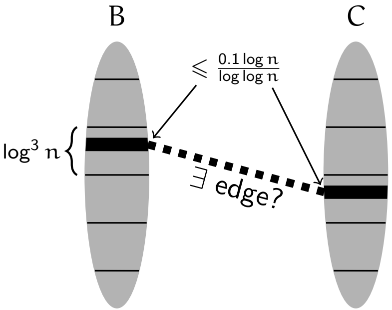

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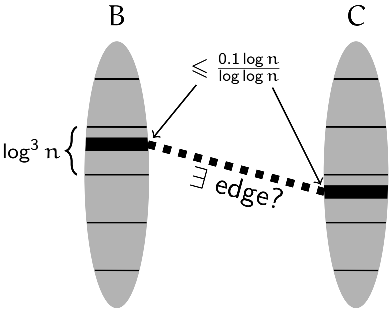

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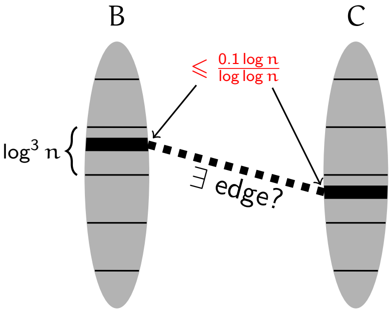

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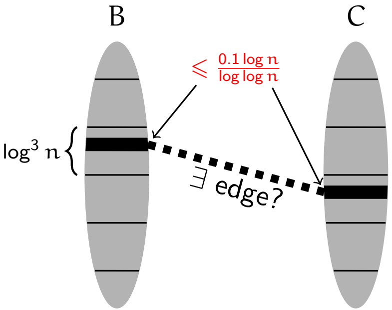

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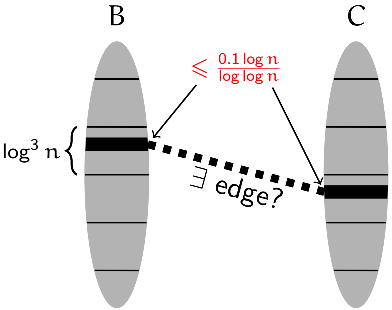
- $\hat{O}(d_{v,B}/\log n + n/\log^3 n)$, $\hat{O}(d_{v,C}/\log n + n/\log^3 n)$ small subsets respectively



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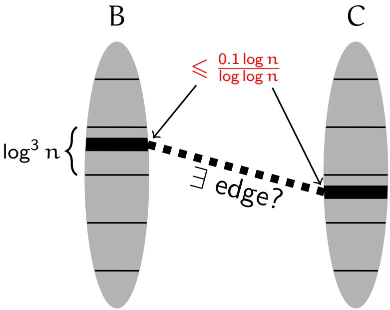

Spend $\hat{O}(n^3/\log^4 n)$ in total.

The Algorithm

Given an n-node tripartite graph, fix $\Delta = \frac{\log n}{100(\log\log n)^2}$.

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Step 1:

Step 2:

Step 3:

Step 4:

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Step 2: Otherwise, find a $v \in A$ such that $d_{v,B} d_{v,C} \geqslant |B||C|/\Delta^2$; let B_v [resp. C_v] be v's neighbourhood in B [resp. C];

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Given an n-node tripartite graph, fix $\Delta = \frac{\log n}{100(\log\log n)^2}$.

Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search;

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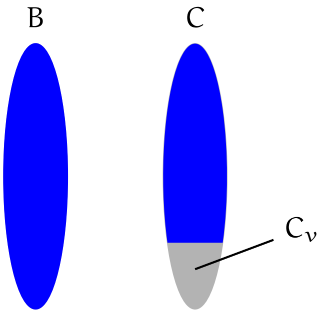

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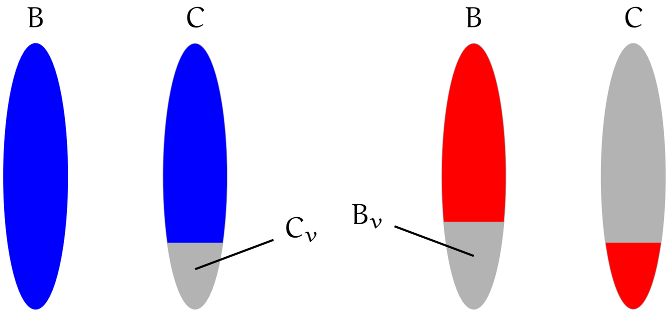

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Correctness: straightforward.

Analysis of the Running Time

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Analysis of the Running Time

Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search; $O(n|B||C|)$

Step 1: If for every $v \in A$, $d_{v,B} d_{v,C} \leqslant |B||C|/\Delta^2$, use the sparse case algorithm;

Step 2: Otherwise, find a $v \in A$ such that $d_{v,B} d_{v,C} \geqslant |B||C|/\Delta^2$; let B_v [resp. C_v] be v's neighbourhood in B [resp. C];

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Analysis of the Running Time

- Small graph case:

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Analysis of the Running Time

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Large Graph Case

Large graph case ($|B|, |C| \geqslant \sqrt{n}$):

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- check degrees, generate inputs for recursion. $O(n(|B| + |C|))$

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Charge the running time to pairs in $B \times C$:

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Charge the running time to pairs in $B \times C$:

- sparse case: charge to $B \times C$ evenly. $\hat{O}(n/\log^4 n)$ per pair

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Every pair is charged at most once!

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Every pair is charged at most once!

- only charge to pairs not going into the same recursive branch.

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Time spent on large graph case: $\hat{O}(n^3/\log^4 n)$.

Analysis of the Running Time

- Small graph case:

Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search; $O(n|B||C|)$

- Large graph case:

Step 1: If for every $v \in A$, $d_{v,B} d_{v,C} \leqslant |B||C|/\Delta^2$, use the sparse case algorithm; $\hat{O}(n|B||C|/\log^4 n)$

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$O(n(|B| + |C|))$

Small Graph Case

Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search; $O(n|B||C|)$

Let v: $d_{v,B} d_{v,C} \geqslant |B||C|/\Delta^2$. $(\Delta = \Theta(\log n/(\log \log n)^2))$

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Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search; $O(n|B||C|)$

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For simplicity, assume $d_{v,B} = |B|/\Delta$, $d_{v,C} = |C|/\Delta$.

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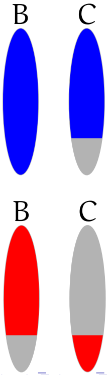

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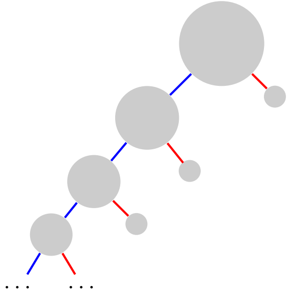

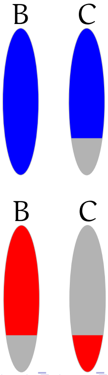

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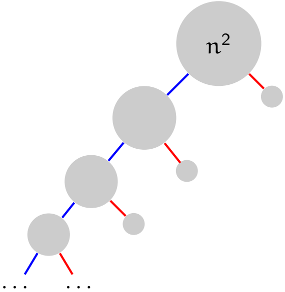

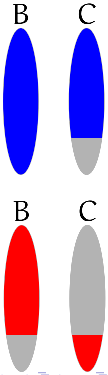

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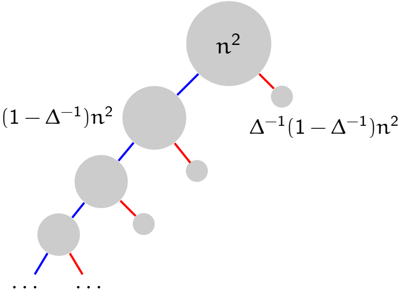

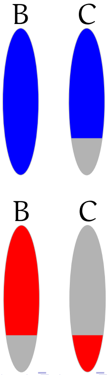

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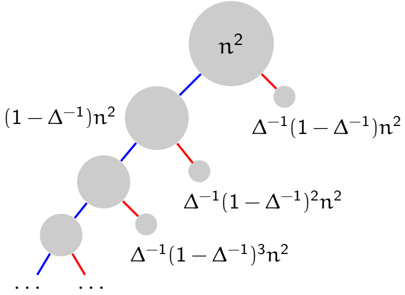

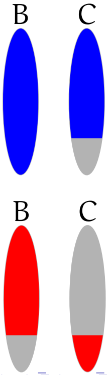

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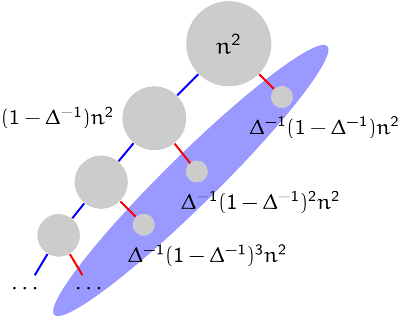

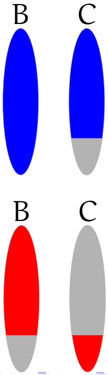

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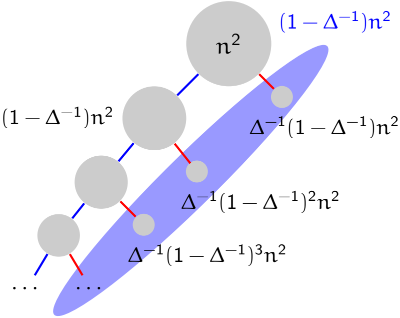

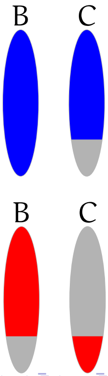

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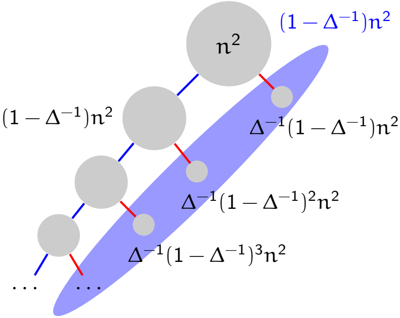

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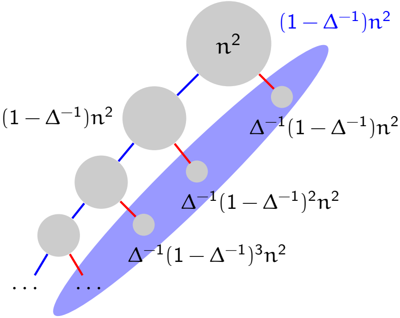

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$\#$ nodes with $O(\Delta \log\log n)$ right steps: $n^{0.4}$

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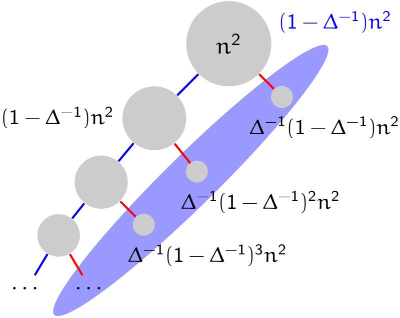

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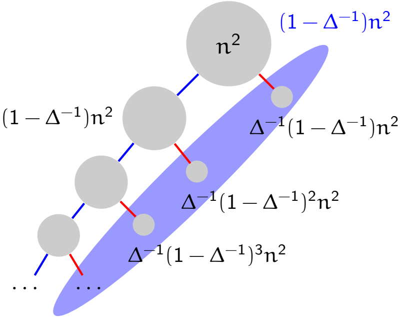

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$O(n^3/\log^{10} n)$ time.

The Algorithm

Given an n-node tripartite graph, fix $\Delta = \frac{\log n}{100(\log\log n)^2}$.

Step 0: If $|B| < \sqrt{n}$ or $|C| < \sqrt{n}$, use exhaustive search;

Step 1: If for every $v \in A$, $d_{v,B} d_{v,C} \leqslant |B||C|/\Delta^2$, use the sparse case algorithm;

Step 2: Otherwise, find a $v \in A$ such that $d_{v,B} d_{v,C} \geqslant |B||C|/\Delta^2$; Let B_v [resp. C_v] be v's neighbourhood in B [resp. C];

Step 3: Check all pairs in $B_v \times C_v$ for an edge;

Step 4: If $|B_v|/|B| > |C_v|/|C|$,
then recurse on $(A, B, C \setminus C_v)$ and $(A, B \setminus B_v, C_v)$,
else recurse on $(A, B \setminus B_v, C)$ and $(A, B_v, C \setminus C_v)$.

Correctness: straightforward.

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Thanks!